A NEW MANIFESTATION OF FLUIDITY PROPERTIES IN CERTAIN FREEFLOWING MATERIALS

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Results are presented for the experimental study of the motion of freeflowing materials in a dense layer [bed] on cylindrical and flat models.

To study the features involved in the motion of free-flowing materials in equipment and bunker installations, we generally make use of models having transparent walls [1–8] or we employ models which can be dismantled [9,10].

Many researchers have noted that even with the so-called second form of discharge [2], when all of the freeflowing material above the EF level (Fig. 1a) is in motion, a "dead" zone M_1 is formed on the flat horizontal bottom of the model, with the particles remaining fixed in this zone. The literature contains contradictory data on the relative dimensions of this zone [1,7,8].

To clarify this problem, we experimented with a cylindrical glass model with an inside diameter of 150 mm, a height of 1700 mm, and a horizontal transparent bottom made of plastic. An aluminosilicate catalyst of the following fractional composition served as the freeflowing material: 45% of the particles with a grain size of 3–4 mm; 51% of the particles with a grain size of 4–5 mm; and 4% of the particles with a grain size of 5–5.5 mm. The particles exhibiting a grain size of 3–4 mm were virtually spherical in shape; the larger particles were oval in shape. The degree of ovality, defined by the ratio of the maximum-to-minimum grain dimensions, amounted, respectively, to 4, 12, and 20% for the indicated fractions. The average bulk weight of the catalyst was 0.78 g/cm^3; the apparent specific weight of the particles was 1.8 g/cm^3; the angle of rest was 28°; according to measurement data obtained with a triaxial compression instrument, the internal friction factor was 0.577.

For the continuous flow of this material through an orifice 18 mm in diameter, we noted that the relative dimensions of the M_1 zone were considerably smaller than indicated in the published works of other researchers [1, 3, 7]. At the bottom, the nonmoving material occupies a peripheral annular zone cb_1 (Fig. 1b) that is 27 mm wide, which makes up less than half the radius of the bottom.

The M_2 zone in which the catalyst particles move very slowly is directly adjacent to the M_1 zone from above and at the side of the wall. Thus with an average theoretical bed velocity of 8 cm/min, the average particle speed across segment b_2b_3 amounts to 0.5 cm/min, with the corresponding figure at the b_1b segment 0.35 cm/min. The results of the experiments that we carried out showed that the relative dimensions of a completely stagnated zone are functions of the

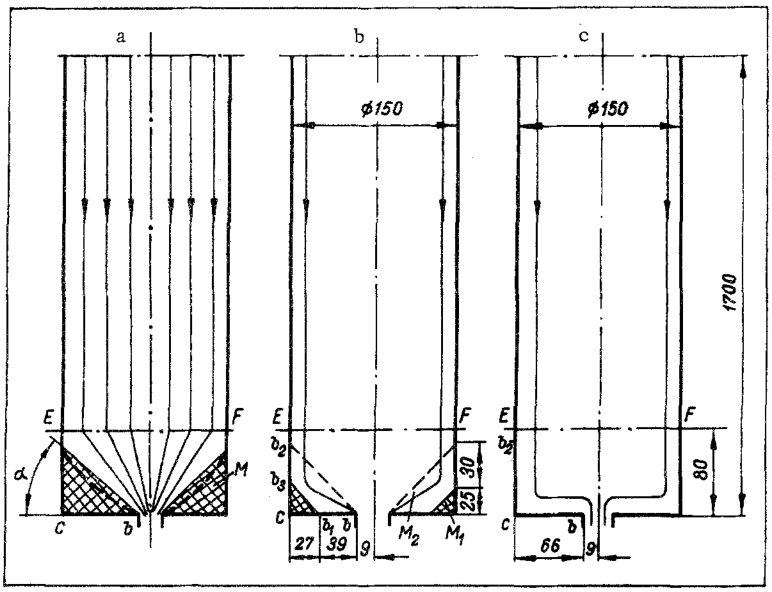

Fig. 1. Particle motion patterns in apparatus and relative sizes of totally stagnant zone: a) According to literature data; b) according to our data for aluminosilicate catalyst; c) according to our data on steel balls.

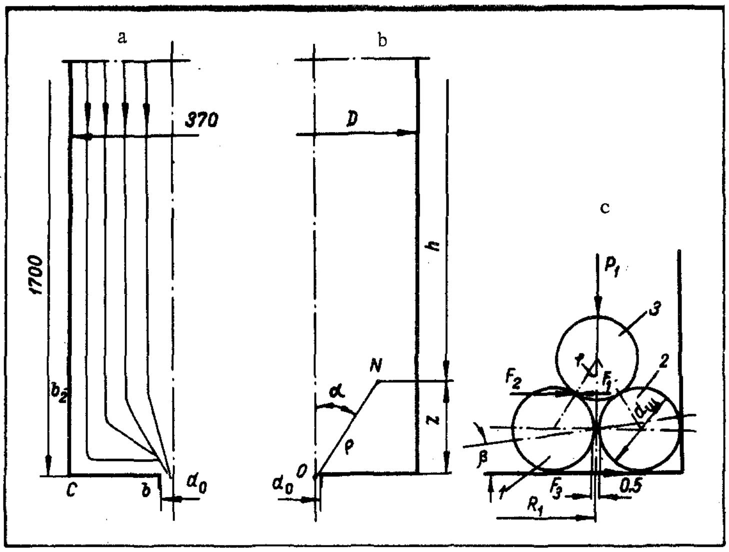

Fig. 2. Trajectories of motion for metallic balls and patterns of voltage redistribution.

degree of fluidity for the freeflowing material, defined by the magnitudes of the internal and external friction factors and the forces of structural adhesion. It had been suggested that in the motion of materials exhibiting even freer fluidity, a further reduction in the

Particle motion (steel balls) over a nonmetallic bottom of a multiped model

Experiment number *;	time (in sec) of particle passage between positions (see Fig. 3)				
	1—2	2—3	3—4	4—5	5—6
16	2820	1205	663	74.0	16.5
17	3035	1360	586	61.1	14.9
18	2978	1176	704	55.8	11.6
19	3456	1290	620	83.3	18.1
20	3385	1408	765	64.6	14.4
21	—	—	—	111.3	24.3
22	—	—	—	93.1	29.1
23	—	—	—	123.1	19.9
24	—	—	—	78.7	23.5
25	—	—	—	89.8	23.8

*In experiments 16-20, the bottom was made of oak; in 21-25, the bottom was made of rubber.

relative dimensions of the M_1 zone should be noted, or even that it will disappear entirely. To test this hypothesis, we experimented with calibrated steel spheres [balls] 4.9 mm in diameter, these being placed into the cited model in a bed 1700 mm high, and then removed from the model through an orifice 18 mm in diameter.

Experiments confirmed the predicted hypothesis: all the balls at the wall in the lower part of the model and on the flat horizontal bottom moved downward very slowly toward the outlet orifice. Thus, for example, with a mean theoretical bed-motion velocity of 6 cm/min, the mean speed of retarded ball motion over the segment b_2c (Fig. 1c) averaged 0.31 cm/min, while the mean velocity of the accelerated motion of the balls over the segment cb was 0.25 cm/min.

In view of the unique nature of the phenomenon of "liquid-like fluidity" of the cited freeflowing medium, we carried out additional experiments on a multilayer [multibed] model for a more detailed study of this phenomenon relative to the conditions of the plane problem.

The multibed model, 1700 mm in height, consisted of two vertical plates made of plastic, separated from each other by a distance of 5.0 mm. The end walls between these plates and the horizontal flat bottom were made of ground steel planks 0.5 mm thick.

The steel balls were fed into the model continuously from a bunker with a capacity of 35 l. At a minimum feed rate of 1500 cm^3/min the bunker made it possible to keep the bed continuously in motion for 23 min.

The experiments carried out on this model also confirmed the basic result of the previous investigations. Figure 2a shows the conditional trajectories of motion for the calibrated balls for an average theoretical bed-motion velocity of 80 cm/min. The mean velocity of ball motion over the segment b_2c does not exceed 4.2 cm/min, while over the segment cb it amounts to 3.4 cm/min.

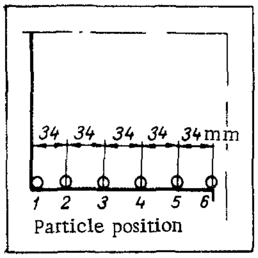

Fig. 3. Diagram for table.

The phenomenon of "liquid-like fluidity" in metallic balls under the indicated conditions can be explained by taking into consideration the redistribution of stresses in the lower zone as a result of the discharge of the freeflowing material through the outlet orifice.

In analogy with the problem on the distribution of stresses in real soil beneath a concentrated load [11], the quantity $\Delta\sigma_\rho$, by which the compressive stress in the direction of the outlet orifice is reduced at an arbitrary point N (Fig. 2b), can be expressed in general form by the function

$$\Delta\sigma_\rho = A(P_i - P_f)\frac{z^{\nu-2}}{\rho^\nu}. \quad (1)$$

It follows from the condition of quasi-static equilibrium for the freeflowing medium that the reduction in the compressive stresses in the direction of the outlet orifice by a quantity $\Delta\sigma_\rho$ is accompanied by an increase in the compressive stresses in the direction perpendicular to the radius ON by the quantity $\Delta\sigma_{fr}$.

The resulting stresses are found by adding the corresponding components $\Delta\sigma_\rho$ and $\Delta\sigma_{fr}$, as well as the compressive stresses produced by the weight of the particles lying above. When $P_f = 0$ (free flow) $D/d_0 = 8$, $\nu = 4$, $h/D = 5$ and Eq. (1) is modified to the form

$$\Delta\sigma_\rho \approx 1.05\cdot10^{-2}\gamma D^3\frac{\cos^2\alpha}{\rho^2}. \quad (2)$$

The selection of the numerical values of D/d_0, h/D and ν in the given example is explained by the following: the ratio $D/d_0 = 8$ is most characteristic of industrial equipment; when $h/D \geq 5$ the vertical pressure due to the weight of the particles lying above assumes a constant value; the minimum value of the distribution-capacity coefficient $\nu = 3$ corresponds to an ideal isotropically elastic medium.

The bed of calibrated steel balls in terms of its distribution capacity is closer to this medium than to a bed of fine-grained sand for which $\nu \approx 6$.

For the balls lying at the bottom of the model near the end wall (Fig. 2c), $\alpha = 90° - \beta = 90° - \operatorname{arctg}\frac{d_s}{2R_1} = 88°$, $\rho = R_1 = 70$ mm. Then when $\gamma = 4.38$ g/cm^3 we find from (2) that

$$\Delta\sigma_\rho = 0.000004 \text{ kg/cm}^2.$$

This quantity is smaller than the vertical stress in the zone under consideration by a factor of approximately 50 000. Evidently, the shifting of the spheres [balls] as a result of this insignificant drop in stress may occur, with the outlet orifice closed, only when the freeflowing medium is in the limit stressed state.

For a continuous medium this condition is expressed by the equation [12]

$$\frac{\sigma_I - \sigma_{III}}{\sigma_I + \sigma_{III}} = \sin\psi, \quad (3)$$

which is satisfied when $\psi \approx 28°$ (a bed of metallic balls) and $\sigma_I/\sigma_{III} \approx 3$ (in accordance with the Rankine hypothesis).

For the particles lying at the bottom of the model it is more convenient to employ the condition of equilibrium based on the scheme of a discrete freeflowing medium, where consideration is given to the frictional force not only between the particles, but also between the particles and the bottom.

Verification of the condition of limit equilibrium may be based on an examination of the forces acting, for example, on sphere 1 (Fig. 2c) in a multibed model for a discrete freeflowing medium.

In accordance with experimental data, the distance between spheres 1 and 2 is assumed to be equal to 0.5 mm, $\varphi = 33°30'$).

The equilibrium equation has the form

$$F_1 = F_2 + F_3. \quad (4)$$

Since the weight of a single ball is considerably smaller than the pressure P_1,

$$F_1 = \frac{1}{2}P_1\operatorname{tg}\varphi, \quad (5)$$

$$F_2 = \frac{1}{2}P_1\mu_1, \quad F_3 = \frac{1}{2}P_1\mu_2. \quad (6)$$

Assuming $\mu_1 = \mu_2 = \mu$, from (3) we find

$$\mu = \frac{1}{2}\operatorname{tg}\varphi = \frac{1}{2}0.66 = 0.33.$$

According to handbook data [13], for dry metal balls $\mu = 0.15$. Thus the shifting of the balls may be observed even when $\mu = 0.33$, if $\Delta\sigma_\rho \neq 0$. Naturally, such movement becomes even more possible when $\mu \leq 0.15$.

The question now arises as to whether the motion is imparted as a result of the fact that the friction between the balls and the bottom is smaller than the friction between the balls themselves.

The answer follows from the results of repeated calculation for $\mu_1 \neq \mu_2$.

In this case Eq. (3) is written in the following form:

$$\frac{1}{2}P_1\operatorname{tg}\varphi = \frac{1}{2}P_1\mu_1 + \frac{1}{2}P_1\mu_2 \quad (7)$$

or $\operatorname{tg}\varphi = \mu_1 + \mu_2$, whence $\mu_2 = \operatorname{tg}\varphi - \mu_1 = 0.66 - 0.15 = 0.51$. Consequently, the motion of the balls at the bottom is possible even with a very high value for the coefficient of friction between the balls and the bottom ($\mu_2 \leq 0.51$). According to the data of [13], $\mu = 0.3 - 0.5$ is assumed for soft steel over hard wood.

To test this conclusion, we carried out experiments in which planks of hard wood were used to cover the bottom of the model in one of the experiments, and rubber plates in the other. The results of these experiments, shown in the table, demonstrate that on a wooden bottom there is absolutely no zone in which the bed is nonmoving, although the velocity of ball motion in the corners of the model is very small and their motion can be observed after a prolonged period of observation.

When the wooden planks are replaced by rubber plates the width of the M zone of the fixed bed increases from 0 to 100 mm. These experiments confirm the results of the cited calculations.

NOTATION

P_i and P_f are the pressure on the area of outlet before and after the beginning of bed motion, respectively; z and ρ are the ordinate and distance of a given point from a discharge center taken as a point 0; ν is the distributability of a real loose medium; A is the coefficient determined from the condition that the value $(P_i - P_f)$ is equal to the sum $\Delta\sigma_\rho$ over all areas with respect to a spherical surface of a radius ρ; γ is the bulk weight of granular material; D is the internal diameter of apparatus; F_1 is the horizontal pressure force on sphere (1); F_2 is the horizontal friction force at a contact point between spheres (1) and (3); F_3 is the friction force at a contact point between sphere (1) and surface of horizontal bottom; P_1 is the pressure force; d_0 is the outlet diameter; h is the height of granular material bed; μ_1 is the friction coefficient between spheres; μ_2 is the friction coefficient between sphere and bottom; σ_1 and σ_{111} are the principal stresses; ψ is the internal friction angle.

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